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# Study of neutron correlations in spontaneous and neutron-induced fission

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**Background** Neutron emission is correlated in fission events because, on average, more than one neutron is emitted per fission. Measurements of these correlations, coupled with studies of more inclusive observables such as neutron multiplicity, provide sensitive information about the fission mechanism. Neutron-neutron correlations have been studied both in spontaneous fission of  $^{252}\text{Cf}$  and neutron-induced fission of  $^{235}\text{U}$ . These correlations, until recently incalculable in most available simulations of fission, can now be calculated in event-by-event simulations of fission.

**Purpose** Phenomenological studies of fission are of interest both for basic science and for practical applications. Neutron-neutron correlations are a characteristic of the fissioning isotope and could be used in material identification.

**Method** We use our model of complete fission events, **FREYA**, to first study the sensitivity of two-neutron correlations to the model inputs and then compare to available data. We also compare our simulations to neutron-fragment angular correlations.

**Results** We find that the correlations calculated with **FREYA** are fairly robust with respect to the input parameters. Any strong deviations in the correlations result in poor agreement with measured inclusive neutron observables such as neutron multiplicity as a function of fragment mass and the neutron multiplicity distribution. The agreement of **FREYA** with the present set of correlation data is found to be good.

**Conclusions** **FREYA** can reliably predict neutron-neutron correlations.

## I. INTRODUCTION

Neutron correlations have been studied since early in the history of fission measurements. Neutron-neutron angular correlations, which can be obtained without the simultaneous measurement of a fission fragment, have been of special interest. Another early observable, which does require the detection of a fragment, is the angular correlation between the emitted neutrons and the light fission fragment,  $\theta_{nL}$ . Taken together, these two observables are sensitive to the characteristics of neutron emission and are thus useful for testing models of fission neutrons.

The first of the neutron-neutron correlation measurements, dating as far back as 1948, was made by DeBenedetti *et al.* [1]. They bombarded a  $^{235}\text{U}$  source with fast neutrons and detected the emitted neutrons by proportional counters placed at various angles around the source. They also made a calibration measurement of the ratio of neutron coincidences at  $90^\circ$  and  $180^\circ$ , trying to account for cross correlations between detectors due to rescattering where a neutron producing a recoil in one counter is scattered into an adjacent detector. To do this, they used a Pb-Be source emitting single neutrons so that all apparent neutron coincidences arise from only one neutron. They assumed that the rate of rescattering coincidences induced by the Pb-Be source also holds for the  $^{235}\text{U}$  source. With this assumption, they found the correlation to be flat for  $\theta_{nn} < 90^\circ$  and increasing above  $90^\circ$ , leading them to conclude that the two neutrons are preferentially emitted from complementary fragments. Because most later experiments have observed an enhancement also around  $\theta_{nn} = 0^\circ$ , they may have overestimated the rescattering background for their source.

Neutrons are usually assumed to be emitted isotropically in the rest frame of the emitting fragment. When boosted to the laboratory frame, where the observations are made, the neutrons thus move preferentially in the same direction as the fragments. Therefore, if one neutron is emitted from each of the two fragments, the correlation exhibits an enhancement around  $\theta_{nn} = 180^\circ$  because the fragments move in opposite directions after scission. On the other hand, if both neutrons come from the same fragment, then they appear preferentially near  $\theta_{nn} = 0^\circ$ . Those are the only neutron sources if one assumes that the neutrons are emitted only from fully-accelerated fragments. However, there is also a possibility that neutrons are emitted during the fission process itself [2–6], though it is not yet clear whether there are in fact such neutrons and, if so, whether they are emitted during the evolution from saddle to scission, as suggested by Kapoor *et al.* [5], during the scission process, or during the subsequent Coulomb acceleration of the fragments, as suggested by Skarsvåg [6]. A number of the later experiments measuring neutron-neutron angular correlations were dedicated to clarifying this issue.

The measured correlations have been simulated assuming the existence of scission neutrons. Thus three neutron sources were considered: the scissioning nucleus and the two fission fragments. The scission neutrons were assumed to be emitted independently and isotropically in the rest frame of the fissioning nucleus. For spontaneous fission this is the laboratory frame, so two scission neutrons exhibit no correlation. This is also the case if one of the two neutrons arises from a fragment and the other is a scission neutron. Consequently, the presence of scission neutrons flattens the two-neutron angular correlation function [7]. Previous simulations of these cor-

relations assumed a 0 – 20% scission contribution to the neutron multiplicity. In addition to the assumption of isotropic emission in the emitter rest frame, the neutron energy spectrum was taken to be the same for all neutrons and independent of the number of neutrons emitted from a given fragment [8, 9].

Another neutron-related angular correlation is that between a neutron and the light fragment. While the identity of the light fragment can be determined with fragment detectors, the source of the detected neutron is unknown. Nonetheless measurements show a strong peak at  $\theta_{nL} = 0^\circ$ . The first such measurements were by Bowman *et al.* [2] and by Skarsvåg and Bergheim [3].

We present here the first calculations of those angular correlations made with a recently developed Monte-Carlo type model that yields samples of complete fission events.

The employed model **FREYA** (Fission Reaction Event Yield Algorithm) incorporates the relevant physics with a few key parameters determined by comparison to data [10–12]. **FREYA** simulates the entire fission process and produces complete fission events with full kinematic information on the emerging fission products and the emitted neutrons and photons, incorporating sequential neutron and photon evaporation from the fission fragments, adjusting the fragment temperature after each emission and conserving the total energy as well as the linear and angular momentum. The event-by-event nature of **FREYA** makes it straightforward to extract the angular correlation between two evaporated neutrons and between an evaporated neutron and the light fission fragment, neither of which can be addressed with the traditional fission models.

In Sec. II we describe the inputs to **FREYA** that could affect these correlations. We then discuss the sensitivity of the neutron-neutron correlation result to these inputs in Sec. III. Section IV compares the calculated results to available data on neutron-neutron correlations (Sec. IV A) and neutron-light fragment correlations (Sec. IV B). Our findings are summarized in Sec. V.

## II. INPUTS

**FREYA** requires a variety of data-based inputs, most importantly the fission yields,  $Y(A_H)$ , and the total fragment kinetic energy,  $\text{TKE}(A_H)$ , associated with the particular energy of the fissioning system. Furthermore, the Gaussian widths of the fragment charge distributions are based on experimental measurements (see Ref. [12]). There are a variety of universal inputs, including ground-state masses (which are taken from data [13] when available and otherwise supplemented by theory [14]); fission barrier heights; and pairing energies and shell corrections. **FREYA** also has several input parameters that may depend on the identity of the fissioning nucleus. These include the shift of the measured TKE required to match the average neutron multiplicity,  $d\text{TKE}$ ; the asymptotic level density parameter,  $e_0$ ; the advantage in excitation

energy given to the light fragment,  $x$ ; the relative thermal fluctuations in the fragment temperature distribution,  $c$ ; the energy above the neutron separation energy where photon emission begins to dominate over neutron emission,  $Q_{\min}$ ; and the ratio of the spin temperature to the scission temperature,  $c_S$ . In the remainder of this section, we introduce these inputs more fully and describe the consequences of varying these inputs on neutron observables.

Once the average total kinetic energy has been specified, the average total fragment excitation energy  $\overline{\text{TXE}}$  follows by energy conservation,

$$\overline{\text{TXE}} = Q + E_0 - \overline{\text{TKE}}, \quad (1)$$

where  $Q = M(A_0, Z_0) - M(A_L, Z_L) - M(A_H, Z_H)$  is the  $Q$ -value for the particular division. **FREYA** employs the back-shifted Fermi gas (BSFG) model in which the level density parameter is given by [15]

$$\tilde{a}_i(E_i^*) = \frac{A_i}{e_0} \left[ 1 + \frac{\delta W_i}{U_i} (1 - e^{-\gamma U_i}) \right], \quad (2)$$

where  $U_i = E_i^* - \Delta_i$  is the effective excitation and  $\gamma = 0.05/\text{MeV}$  is the rate at which the shell effects attenuate with energy [16]. The pairing energy of the fragment,  $\Delta_i$ , and its shell correction,  $\delta W_i$ , are tabulated in Ref. [17] based on the mass formula of Koura *et al.* [18]. We note that if the shell correction is negligible,  $\delta W_i \ll U_i$ , or the available energy is large,  $\gamma U_i \gg 1$ , then the above renormalization is immaterial and the BSFG level density parameter takes on its macroscopic form,  $\tilde{a}_i \approx A_i/e_0$ , in which it is proportional to the mass number. We take  $e_0$  to be an adjustable model parameter. In Ref. [12], we found  $e_0 \approx 10/\text{MeV}$  which we use in these studies for all fissile actinides. To test the influence of  $e_0$  on the neutron correlations, we will vary  $e_0$  by 20%, between 8/MeV and 12/MeV.

If the two fragments are in mutual thermal equilibrium,  $T_L = T_H = T_{\text{sciss}}$ , the total excitation energy will, on average, be partitioned in proportion to the respective heat capacities which in turn are proportional to the level density parameters, *i.e.*  $\overline{E}_i^* \sim \tilde{a}_i$ . **FREYA** first assigns tentative average fragment excitation energies based on such an equipartition,

$$\dot{E}_i^* = \frac{\tilde{a}_i(\tilde{E}_i^*)}{\tilde{a}_L(\tilde{E}_L^*) + \tilde{a}_H(\tilde{E}_H^*)} \overline{\text{TXE}}, \quad (3)$$

where  $\tilde{E}_i^* = (A_i/A_0)\overline{\text{TXE}}$ . Subsequently, because the observed neutron multiplicities suggest that the light fragments tends to be disproportionately excited, the average values are adjusted in favor of the light fragment,

$$\overline{E}_L^* = x \dot{E}_L^*, \quad \overline{E}_H^* = \overline{\text{TXE}} - \overline{E}_L^*, \quad (4)$$

where  $x$  is an adjustable model parameter expected be larger than unity. We have found that  $x = 1.3$  leads to good agreement with  $\nu(A)$  for  $^{252}\text{Cf}(\text{sf})$ , while  $x = 1.2$  is

suitable for  $^{235}\text{U}(n,f)$  [19]. To test the importance of  $x$  for the correlation observables, we will vary  $x$  for  $^{252}\text{Cf}$  by  $\sim 30\%$ , between 1 and 1.6. We also test the effect of taking  $x < 1$  by using  $x = 0.75$ . We will demonstrate that this parameter significantly affects the calculated  $\nu(A)$ .

After the mean excitation energies have been assigned, **FREYA** considers the effect of thermal fluctuations. The mean excitation of a fragment is related to its temperature  $T_i$  by  $\overline{E}_i^* = \tilde{a}_i T_i^2$  and the associated variance in the excitation is  $\sigma_{E_i}^2 = -\partial^2 \ln \rho_i(E_i) / \partial E_i^2 = 2\overline{E}_i^* T_i$ . Therefore, for each of the two fragments, a thermal energy fluctuation  $\delta E_i^*$  is sampled from a Gaussian distribution of variance  $2c\overline{E}_i^* T_i$  where the parameter  $c$  makes it possible to test the sensitivity of the observables to the magnitude of the energy fluctuations. The fragment excitations are modified accordingly, yielding

$$E_i^* = \overline{E}_i^* + \delta E_i^*, \quad i = L, H. \quad (5)$$

Due to energy conservation, there is a compensating opposite fluctuation in the total kinetic energy,

$$\text{TKE} = \overline{\text{TKE}} - \delta E_L^* - \delta E_H^*. \quad (6)$$

The factor  $c$  multiplying the variance can, in principle, be tuned to the neutron multiplicity distribution  $P(\nu)$ . As a default value, used in our previous work, we take  $c = 1.0$ . We vary  $c$  by 20%, between 0.8 and 1.2. Since  $P(\nu)$  is sensitive to this quantity, we will show the effect of changing  $c$  on  $P(\nu)$  for  $^{252}\text{Cf}(\text{sf})$ .

**FREYA** carries out sequential neutron evaporation until the fragment excitation energy  $E_i^*$  has fallen below  $S_n + Q_{\min}$ , where  $S_n$  is the neutron separation energy  $S_n = M(^{A_i}Z_i) - M(^{A_i-1}Z_i) - m_n$ . It is thus assumed that  $\Gamma_n \gg \Gamma_\gamma$  above that threshold while  $\Gamma_n \ll \Gamma_\gamma$  below it. As a default, **FREYA** uses  $Q_{\min} = 0.01$  MeV so that neutron evaporation continues as long as it is energetically possible. We will see how the correlations are affected if the value is raised to 1 MeV.

In Ref. [20, 21] we included angular momentum in **FREYA**. Generally (though not for spontaneous fission), the fissioning nucleus may have some initial angular momentum. Furthermore, in addition to the resulting overall rotation of the system, part of which is inherited by the fragments, the fragments may acquire fluctuating amounts of additional angular momentum during scission. **FREYA** includes this possibility by assuming that the dinuclear wriggling ( $s_+$ ) and bending ( $s_-$ ) modes (in which the fragments rotate in the same or in the opposite sense around an axis perpendicular to the dinuclear axis, respectively) are being populated statistically (whereas the tilting and twisting modes are assumed to be unimportant). Thus, in each event, the values of  $s_\pm = (s_\pm^x, s_\pm^y, 0)$  are sampled from distributions of the form

$$P_\pm(s_\pm = (s_\pm^x, s_\pm^y, 0)) ds_\pm^x ds_\pm^y \sim e^{-s_\pm^2/2\mathcal{I}_\pm T_S} ds_\pm^x ds_\pm^y, \quad (7)$$

where  $\mathcal{I}_\pm$  are the moments of inertia and the spin temperature is given by  $T_S = c_S T_{\text{sciss}}$ . The coefficient  $c_S$  is regarded as a global but somewhat adjustable model parameter. We use  $c_S = 1$  as a default because physical arguments suggest that it should be close to unity. This value yields rather good agreement with the average energy of photons emitted in fission and gives, for example,  $\overline{S}_L \approx 6.2\hbar$  and  $\overline{S}_H \approx 7.6\hbar$  for  $^{252}\text{Cf}(\text{sf})$ . We have also used  $c_S = 0.1$  as an alternative to dial down the photon multiplicity. For  $^{252}\text{Cf}(\text{sf})$  we then get  $\overline{S}_L \approx 1.8\hbar$  and  $\overline{S}_H \approx 2.2\hbar$  and there are hardly any yrast photons (see Ref. [21] for details).

The moments of inertia of the wriggling and bending modes,  $\mathcal{I}_\pm$ , are given in terms of the moments of inertia of the individual fragments,  $\mathcal{I}_L$  and  $\mathcal{I}_H$ , as well as the moment of inertia associated with the relative fragment motion,  $\mathcal{I}_R = \mu R^2$ . For the former we use  $\mathcal{I}_i = c_I \frac{1}{5} M_i R_i^2$ , i.e. we reduce the rigid-body value by the factor of  $c_I = 0.5$  [20, 21], as is commonly done; we do not vary it here because it affects only the photon observables.

### III. SENSITIVITY OF NEUTRON-NEUTRON ANGULAR CORRELATIONS TO INPUTS

We first study the robustness of the correlation observables by changing the input parameters one at a time from their default values of  $x = 1.3$ ,  $e_0 = 10/\text{MeV}$ ,  $c_S = 1$ ,  $Q_{\min} = 0.01$  MeV and  $c = 1$  for  $^{252}\text{Cf}(\text{sf})$ . We do not change  $d\text{TKE}$  from its value of 0.5 MeV with  $c_S = 1$  because most of the changes we make do not strongly affect the calculated  $\overline{\nu}$ . Nor do we change  $c_I$  controlling the moments of inertia because they affect only the photon observables, whereas the effect on the neutron observables is negligible. The results for the neutron-neutron correlation function are shown in Fig. 1 for  $^{252}\text{Cf}(\text{sf})$ . We consider  $^{252}\text{Cf}(\text{sf})$  because these correlations have been studied most for this system. We employ a minimum neutron energy of  $E_n = 0.5$  MeV for both emitted neutrons. Increasing the minimum neutron energy tends to enhance the correlation near  $\theta_{nn} = 0^\circ$  while giving only a negligible change around  $\theta_{nn} = 180^\circ$ , see Ref. [19].

Figure 1(a) shows the sensitivity of the correlation to  $c_S$  and  $Q_{\min}$ , the parameters most closely related to the photon observables. Changing  $c_S$  from 1 to 0.1 reduces the initial spin from  $\sim 7\hbar$  to  $\sim 2\hbar$ . The default value,  $c_S = 1$ , is most compatible with previous extractions of fragment spins at scission [21]. The two calculations effectively coincide, thus the correlation is insensitive to this parameter. The default cutoff energy for neutron emission,  $Q_{\min} = 0.01$  MeV, effectively allows neutron emission all the way down to the threshold (given by the separation energy  $\sim S_n$ ). Increasing it to 1 MeV has only a small effect on the correlation, reducing the value at  $\theta_{nn} = 180^\circ$  somewhat and making the correlation slightly more symmetric around  $\theta_{nn} = 90^\circ$ . The higher value of  $Q_{\min}$  is more compatible with the energy spectra of photon emission, see Ref. [20]. Therefore,

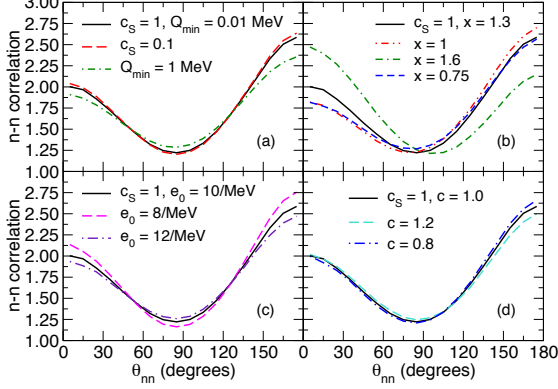


FIG. 1: (Color online) The correlation between two neutrons emitted from  $^{252}\text{Cf(sf)}$  as a function of the opening angle between the two neutrons,  $\theta_{nn}$ . The FREYA results are shown for neutron kinetic energies  $E_n > 0.5$  MeV. The results of different parameter choices in FREYA are compared to the results of the default values:  $c_S = 1$ ,  $Q_{\min} = 0.01$  MeV,  $x = 1.3$ ,  $e_0 = 10/\text{MeV}$ , and  $c = 1$  (solid black curve). (a) Parameters affecting photon emission are varied, reducing the spin temperature,  $c_S = 0.1$  (dashed red curve), or increasing the evaporation cut-off,  $Q_{\min} = 1$  MeV (dot-dashed green curve). (b) The parameter  $x$  controlling the temperature imbalance between the fragments is varied, either making the light fragment even hotter than in the default scenario,  $x = 1.6$  (dot-dashed-dashed green curve), eliminating the imbalance,  $x = 1$  (red curve), or making the heavy fragment the hotter one,  $x = 0.75$  (dashed blue curve). (c) The parameter  $e_0$  governing the level density is varied, either decreasing it to  $e_0 = 8$  MeV (dashed magenta curve) or increasing it to  $e_0 = 12$  MeV (dot-dashed maroon curve). (d) The parameter  $c$  governing the thermal fluctuations of the fragment excitations is either increased to  $c = 1.2$  (dashed turquoise curve) or decreased to  $c = 0.8$  (dot-dashed blue).

while these parameters do not have a strong effect on the neutron correlation, they do influence the photon observables significantly.

The most striking effect on the shape of the neutron-neutron correlation is caused by varying  $x$  (which governs the partition of the excitation energy between the light and heavy fragments). Changing  $x$  while keeping it above unity has a somewhat larger effect at  $\theta_{nn} = 0^\circ$  than changing  $Q_{\min}$  but the effect at  $\theta_{nn} = 180^\circ$  is small. As long as the light fragment is hotter than the heavy fragment, the correlation is largely unaffected. Even though the zero-degree correlation is stronger for neutrons emitted by the light fragment, due to its larger velocity, making it even hotter does not strongly alter the correlation. However, reducing  $x$  below unity, thus making the heavy fragment the hotter one, essentially tilts the correlation function, making it more symmetric around  $\theta_{nn} = 90^\circ$ . Such a large change in  $x$  should affect other observables noticeably which may help to constrain this parameter.

In Fig. 2, we show  $\nu(A)$  for the same values of  $x$  as

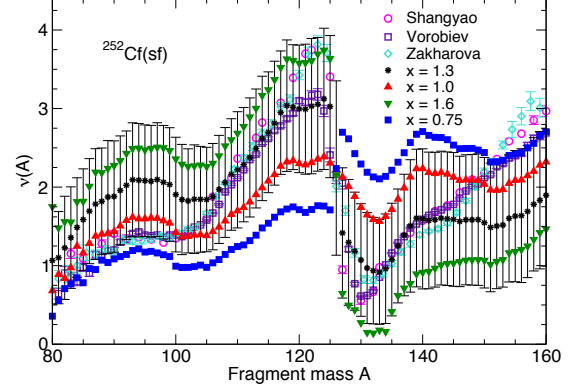


FIG. 2: (Color online) The variation of  $\nu(A)$  with the parameter  $x$ , governing the partition of the excitation energy between the two fragments, is compared to data [22–24]. The default result (black dots) shows bars representing the dispersion in the neutron multiplicity for fragments with the specified  $A$ . The results for  $x = 1$  (solid red, upward triangles),  $x = 1.6$  (solid green, downward facing triangles) and  $x = 0.75$  (blue squares) do not show the dispersions.

in Fig. 1(b),  $x = 0.75, 1, 1.3$  and  $1.6$ . For the default value of  $x = 1.3$  we also show the dispersion in the neutron multiplicity  $\nu$  for fragments with the specified mass number  $A$ . We also show several sets of recent data which agree well with each other. The agreement of our default calculations with these data is quite good for  $105 < A < 145$ , covering the symmetric region and the  $A$  range where the yields are highest. To improve the overall agreement, we would have to introduce an  $A$ -dependent temperature distribution, as has been done in some other calculations [25, 26] or use point-by-point yields for each fragment pair [27]. Increasing  $x$  to 1.6 increases  $\nu(A)$  for the light fragment to well above the data while underestimating the neutron multiplicity for the heavy fragment. It enhances the difference in neutron emission for  $120 \leq A \leq 132$ . Using  $x = 1$  decreases the variation of  $\nu(A)$  considerably. While this actually improves agreement with the data for  $A < 100$ , the edge of the sawtooth is not sufficiently sharp. Finally,  $x = 0.75$  inverts the sawtooth shape, as one would expect, significantly underestimating the yield below symmetry while overestimating the neutron multiplicity above symmetry. Thus while we can see some dependence of the correlation function on  $x$ , these variations can be ruled out by the data on  $\nu(A)$ .

Next, we show the dependence of the neutron-neutron correlation on  $e_0$ , governing the asymptotic value of the level density parameter, in Fig. 1(c). Changing  $e_0$  by  $\pm 2/\text{MeV}$  modifies the correlation somewhat but generally less than changing  $Q_{\min}$  in Fig. 1(a). While the chosen range of  $e_0$  is within the range of acceptable values, it is constrained by the shape of the prompt fission

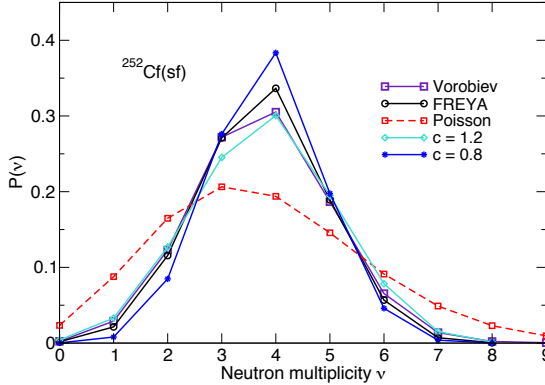


FIG. 3: (Color online) The variation in the neutron multiplicity distribution obtained from varying the thermal fluctuations,  $c$ . The data [28] (violet squares) are compared to the default (labeled FREYA) result as well as that with  $c = 1.2$  (turquoise diamonds) and  $c = 0.8$  (blue squares). The red dashed curve shows the Poisson distribution that has the same average multiplicity.

neutron spectrum, see *e.g.* Ref. [12].

Finally, we illustrate the sensitivity of the FREYA results to the variance of the thermal fluctuations in Fig. 1(d). These fluctuations can modify the intrinsic excitation energy and thus the neutron multiplicity distribution,  $P(\nu)$ , see Fig. 3. While changing  $c$  has a negligible effect on the correlation function, it has a stronger effect on  $P(\nu)$ . Increasing  $c$  to 1.2 broadens  $P(\nu)$  relative to the data, while decreasing it to 0.8 narrows the multiplicity distribution relative to the data. Thus significant changes of  $c$  can be ruled out.

#### IV. COMPARISON TO DATA

Here we take our default values for FREYA, shown as the solid black curve in Fig. 1, and compare to available data. We first compare to existing neutron-neutron correlation data in Sec. IV A. We focus first on  $^{252}\text{Cf(sf)}$  data from Pringle and Brooks [8] and then on more recent data from Gagarski *et al.* [7]. Our calculations have also been compared to the recent  $^{252}\text{Cf(sf)}$  data of Pozzi *et al.* and appear in Ref. [29]. Therefore we do not show them again here but rather refer the reader to that work. We also compare to the  $^{235}\text{U}(n_{\text{th}}, f)$  data of Franklyn *et al.* [9]. In Sec. IV B, we compare our calculations of neutron-light-fragment angular correlations to  $^{252}\text{Cf(sf)}$  data from Bowman *et al.* [2] and from Skarsvåg and Bergheim [3]. Some of these early data have also been compared to Monte Carlo studies albeit not with complete events [30].

##### A. Neutron-Neutron Correlation Data

As we have already discussed, prompt neutrons from fission tend to be either forward or backward correlated. Because the possibility of scission neutrons is not considered, there are three different cases: both neutrons detected were emitted by the light fragment, both were emitted by the heavy fragment, and one neutron was emitted by each fragment. In Ref. [31] we analyzed  $^{239}\text{Pu}(n_{\text{th}}, f)$  for  $\nu = 2$  and found a significant correlation around  $\theta_{nn} = 0^\circ$  when both neutrons are emitted from the same fragment, with a higher peak when that fragment is the lighter one, due to its higher velocity. On the other hand, when one neutron is emitted from each fragment, there is an enhancement around  $\theta_{nn} = 180^\circ$ . The overall result is a stronger backward correlation because emission from both fragments is more likely than emission from the same fragment. This is especially true when the total neutron multiplicity is low and the backward correlation is then strongest. Large multiplicities generally reduce the angular correlations [19].

As we will see, the agreement of our calculations with the data is quite good and there appears to be no compelling need for scission neutrons.

##### 1. $^{252}\text{Cf(sf)}$

An early neutron-neutron correlation measurement was performed by Pringle and Brooks in 1975 [8]. They used two liquid scintillator neutron detectors and employed pulse-shape discrimination to reject photon events. One detector was fixed in the horizontal plane with the source while the second detector was rotated around the vertical axis. At  $\theta_{nn} < 45^\circ$  the distance between the detector and the source was increased and shielding was inserted to reduce neutron rescattering. The minimum detected neutron energy was 0.7 MeV [8].

The more recent measurement by Gagarski *et al.* [7] was published in 2008. They used a similar setup of two neutron detectors with varying angular difference around the source. The detectors were stilbene crystals with photomultiplier tubes surrounded by shielding. They used time-of-flight to separate neutrons from photons. They showed that they could achieve neutron and photon separation with the photomultiplier tubes down to the detector threshold. By changing the event selection boundaries, they were able to employ several different neutron detection thresholds: 0.425, 0.55, 0.75, 0.8, 1.2 and 1.6 MeV [7].

The Gagarski measurement for the 0.75 MeV neutron threshold was compared to the Pringle and Brooks measurement at 0.7 MeV in Ref. [7]. They found relatively good agreement between the two measurements at  $\theta_{nn} < 90^\circ$  but the Gagarski result shows a stronger back-to-back correlation than that of Pringle and Brooks, see Fig. 4. The difference between the two measurements was noted in Ref. [7] but no reason for the discrepancy was

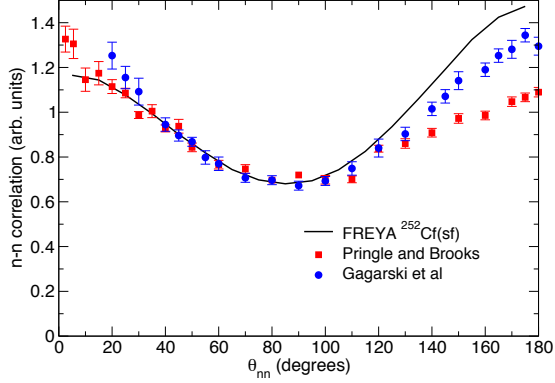


FIG. 4: (Color online) The default FREYA calculations are compared to the  $^{252}\text{Cf}(\text{sf})$  two-neutron correlation data from Pringle and Brooks [8] (red squares) and from Gagarski *et al.* [7] (blue circles) for neutron kinetic energies above 0.7 MeV.

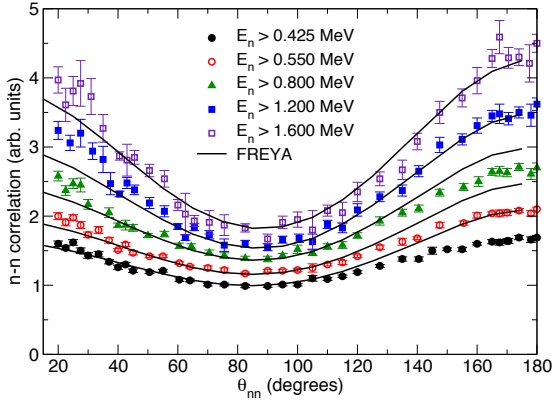


FIG. 5: (Color online) The default FREYA calculations are compared to the  $^{252}\text{Cf}(\text{sf})$  two-neutron correlation data from Gagarski *et al.* [7] for neutron kinetic energies greater than 0.425, 0.55, 0.8, 1.2 and 1.6 MeV.

proposed. The 0.05 MeV difference in energy thresholds is too small to account for it.

Figure 4 also shows the FREYA result. The calculation agrees well with both data sets at  $\theta_{nn} < 90^\circ$  but overestimates the back-to-back correlation at larger angles. Our calculation is relatively close to the Gagarski result although it is slightly above. Given that increasing  $Q_{\min}$  was seen to decrease the calculated correlation at  $\theta_{nn} \approx 180^\circ$ , using a higher  $Q_{\min}$  would improve our agreement with the data.

In Fig. 5 our results are compared to the data from Gagarski *et al.* using their other energy thresholds. We again see that the agreement between the calculation and the data is very good for  $\theta_{nn} < 90^\circ$  while the calculation overestimates the back-to-back peak for neutrons with

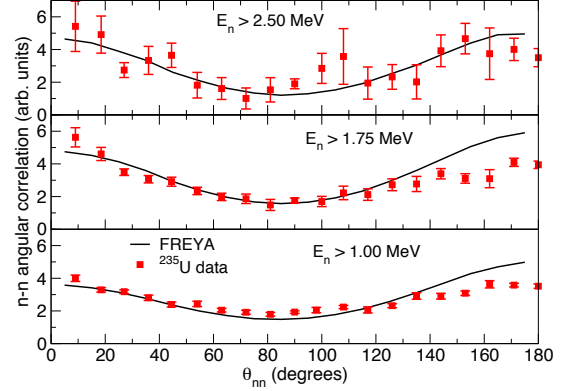


FIG. 6: (Color online) The default FREYA calculations are compared to the  $^{235}\text{U}(n_{\text{th}}, f)$  two-neutron correlation data from Franklyn *et al.* [9] for neutron kinetic energies above 1.0 MeV (bottom), 1.75 MeV (center) and 2.5 MeV (top).

kinetic energies less than 1 MeV. The improvement of the agreement between the calculations and the data at higher neutron energy thresholds suggests that the dependence of the correlation on  $Q_{\min}$  may diminish with neutron energy.

$$2. \quad ^{235}\text{U}(n, f)$$

In 1978 Franklyn, Hofmeyer and Mingay [9] studied neutron-neutron correlations in  $^{235}\text{U}(n_{\text{th}}, f)$ . They used two stilbene neutron detectors and employed pulse-shape discrimination to reject photon events. Boron-loaded shadow shields were used to suppress neutron rescattering effects at low  $\theta_{nn}$ . They obtained correlation results for minimum neutron energies of 1.0, 1.75 and 2.5 MeV. The lower limit on the neutron kinetic energy was imposed by the pulse-shape discriminator [9].

These data are compared to our default FREYA calculations for  $^{235}\text{U}(n_{\text{th}}, f)$  in Fig. 6. The agreement is generally rather good for  $\theta_{nn} < 140^\circ$  with  $E_n \geq 1.0$  MeV and  $E_n \geq 1.75$  MeV though our results again overestimate the back-to-back correlation somewhat. For  $E_n \geq 2.5$  MeV, the agreement is good over all  $\theta_{nn}$ , again suggesting that perhaps a larger value of  $Q_{\min}$  should be employed.

## B. Neutron-Light-Fragment Angular Correlations

In 1962 Bowman *et al.* made the first measurement of correlations between neutrons and light fragments [2]. Their setup consisted of two neutron detectors and two fission fragment detectors, both plastic scintillators of different thickness, mounted around a steel drum of 2 m diameter. A  $^{252}\text{Cf}(\text{sf})$  source was placed at the center of the drum and put under vacuum. The fragment detec-

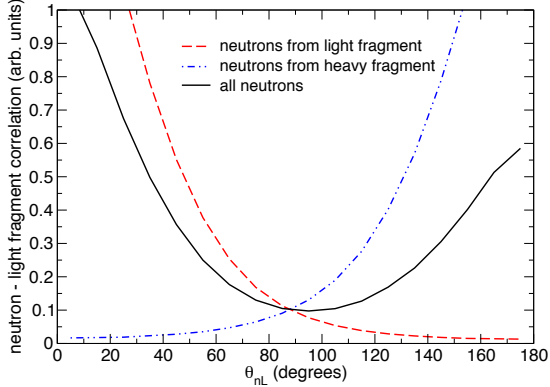


FIG. 7: (Color online) The correlation between the neutrons and the light fragment for  $^{252}\text{Cf}(\text{sf})$  as calculated with **FREYA** using the default parameter values. The contributions from neutrons emitted the light fragment or from the heavy fragment are shown separately. All contributing neutrons have a minimum kinetic energy of 0.5 MeV.

tors were mounted on opposite sides of the drum, at  $180^\circ$  from each other. One neutron detector was held fixed at  $\theta_{nn} = 11.25^\circ$  while the other was moved through angles  $22.5^\circ$  to  $90^\circ$  with respect to one fragment detector ( $111.5^\circ$  to  $180^\circ$  relative to the other fragment detector). Time-of-flight was used to detect one neutron in coincidence with two fission fragments as well as to separate the light and heavy fragments from each other. They presented the angular correlation between all measured neutrons and the identified light fragment. While the correlation is made with the light fragment, it was not possible to determine which fragment emitted the neutron [2].

In Fig. 7 we show how the neutron-light-fragment correlation is built up in **FREYA**. The dashed curve shows the result if all detected neutrons come from the light fragment. There is a strong peak at  $\theta_{nL} = 0$  with essentially no signal in the opposite direction. If all detected neutrons arise from the heavy fragment, the correlation is essentially reflected around  $\theta_{nL} = 90^\circ$ . The shape of the resulting overall correlation retains the largest peak at zero degrees while, in the backward direction, the signal is reduced. This is because more neutrons are emitted by the light fragment due to its initially higher temperature.

In Fig. 8 we compare our **FREYA** results to the measurements of Bowman *et al.* [2] as well as those of Skarsvåg and Bergheim [3] (reproduced in Gagarski *et al.* [7]). The

agreement of the calculated correlation with the shape of the data is very good.

## V. SUMMARY

We have shown that event-by-event models of fission, such as **FREYA**, provide a powerful tool for studying fission

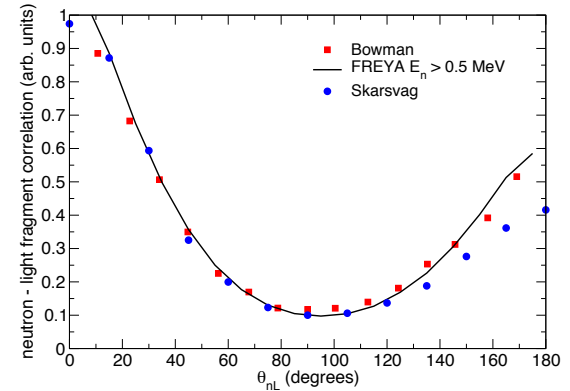


FIG. 8: (Color online) The default **FREYA** calculations are compared to  $^{252}\text{Cf}(\text{sf})$  neutron-light fragment correlation data from [2] (red squares) and [3] (blue circles). The minimum kinetic energy of the neutrons is 0.5 MeV.

neutron correlations. The calculated results are robust, being relatively insensitive to the input parameters which can be constrained by other data. The agreement of our calculations with the available data is good and does not lend strong support for the requirement of scission neutrons to explain the correlations. However, further data on these correlations based on fission of other isotopes and, for neutron-induced fission, at higher incident neutron energies would be valuable for verifying these results.

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